

Dynamic Effects in Superconducting Magnets

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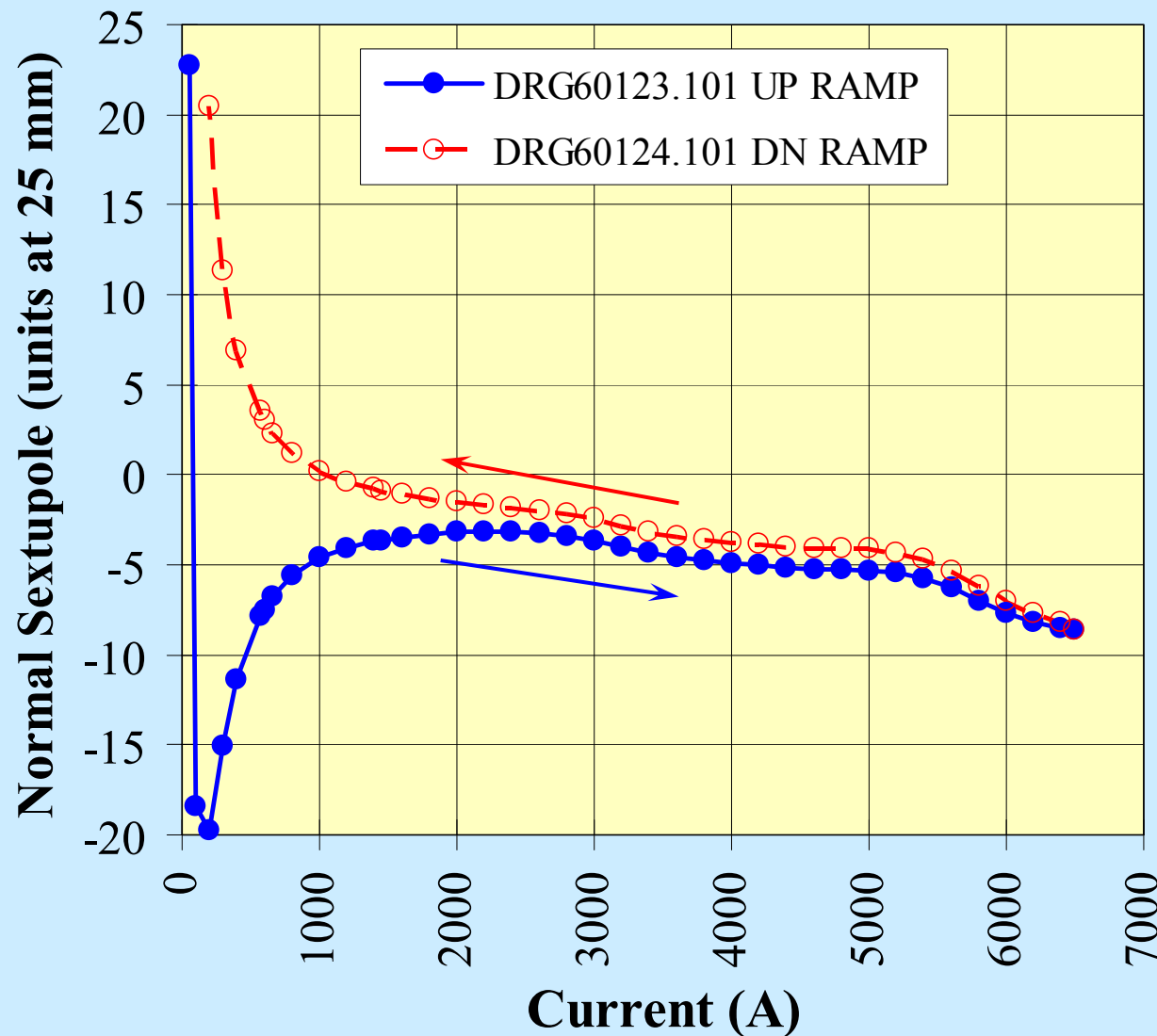
Introduction

- Any change in the field near a superconductor induces eddy currents. These eddy currents are of a persistent nature due to zero resistance in the superconductor.
- Such eddy currents produce field distortions (harmonics) depending on several factors, such as superconductor properties, ramp direction, ramp rate, etc.
- An understanding of these effects is important in the measurements of superconducting magnets.

Hysteresis in Harmonics

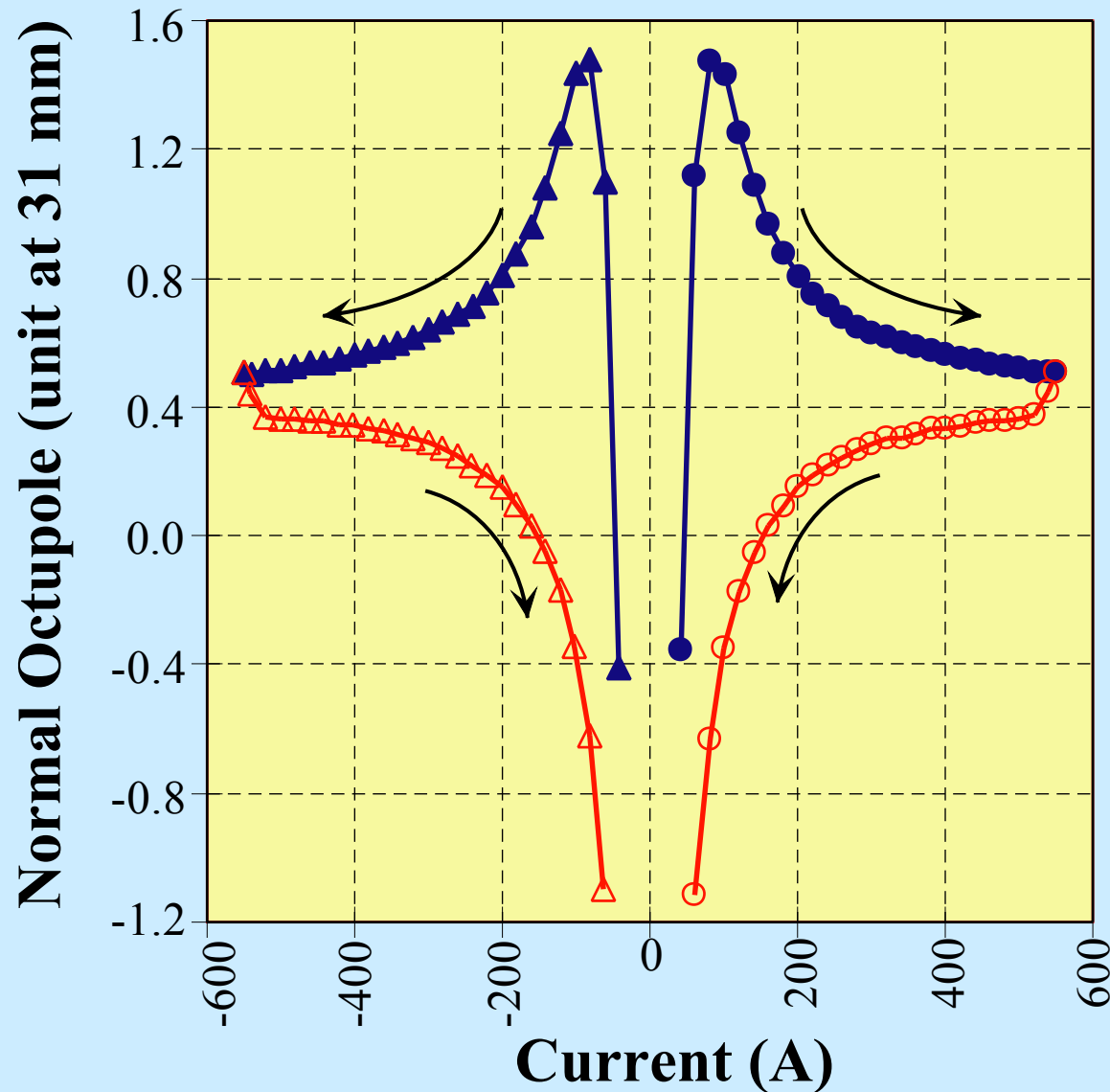
- The dependence of eddy currents on the ramp direction produces a ***hysteresis*** in the harmonics.
- Generally, the eddy currents have symmetries similar to the main field harmonic. Thus, only the allowed harmonics are commonly affected.
- For multilayer magnets with several multipoles, (e.g. corrector packages) the external field may have a symmetry different from the main field. In this case, hysteresis may be seen in some unallowed harmonics also.

Hysteresis in an Allowed Term



In a dipole magnet, a large negative sextupole is produced at low fields on the UP ramp. This changes to a large positive sextupole on the DOWN ramp.

Hysteresis in an Unallowed Term

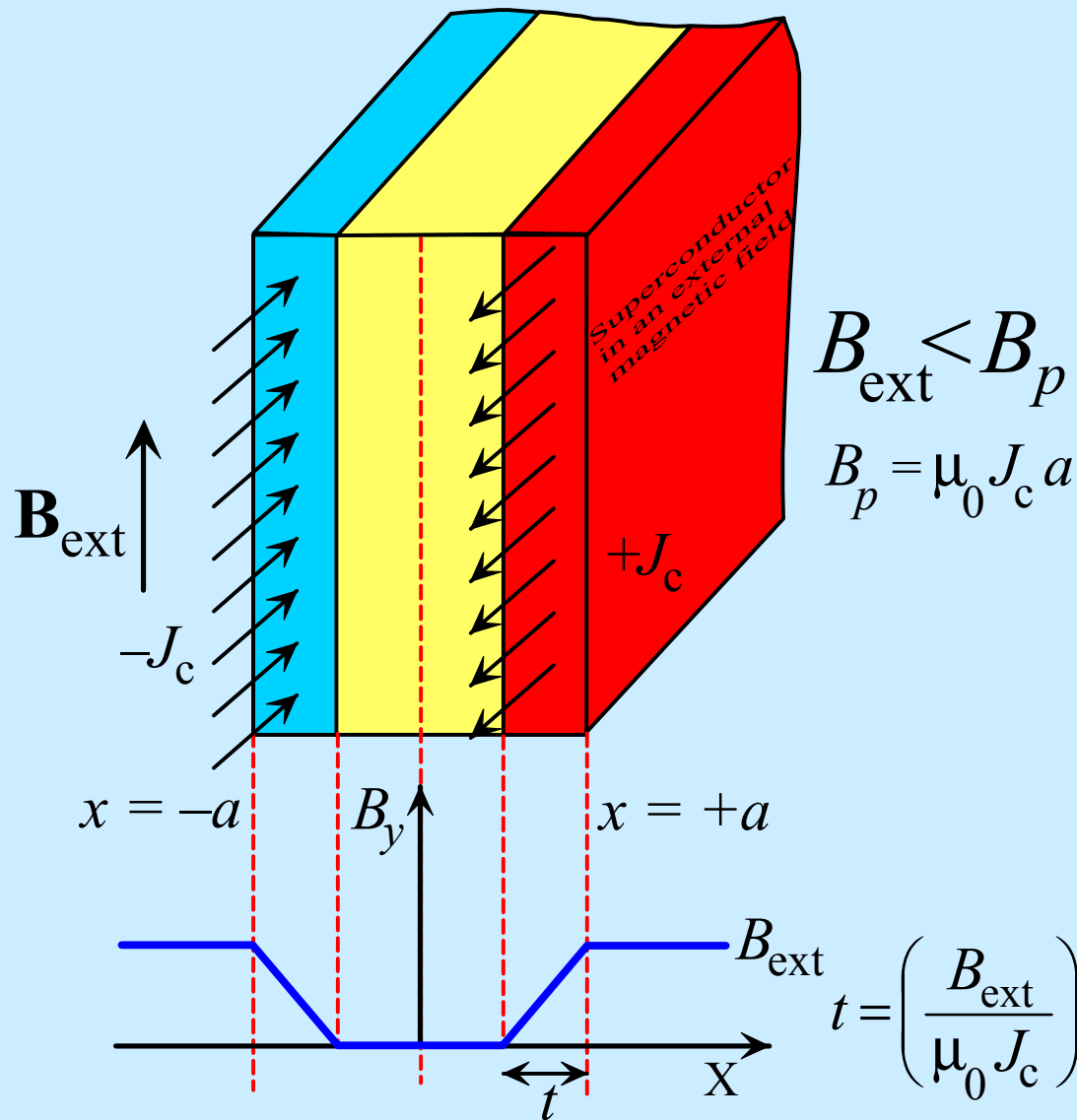


Hysteresis in the octupole term in the quadrupole layer of a multi-layer magnet consisting of several concentric layers of normal and skew multipole magnets.

Critical State Model

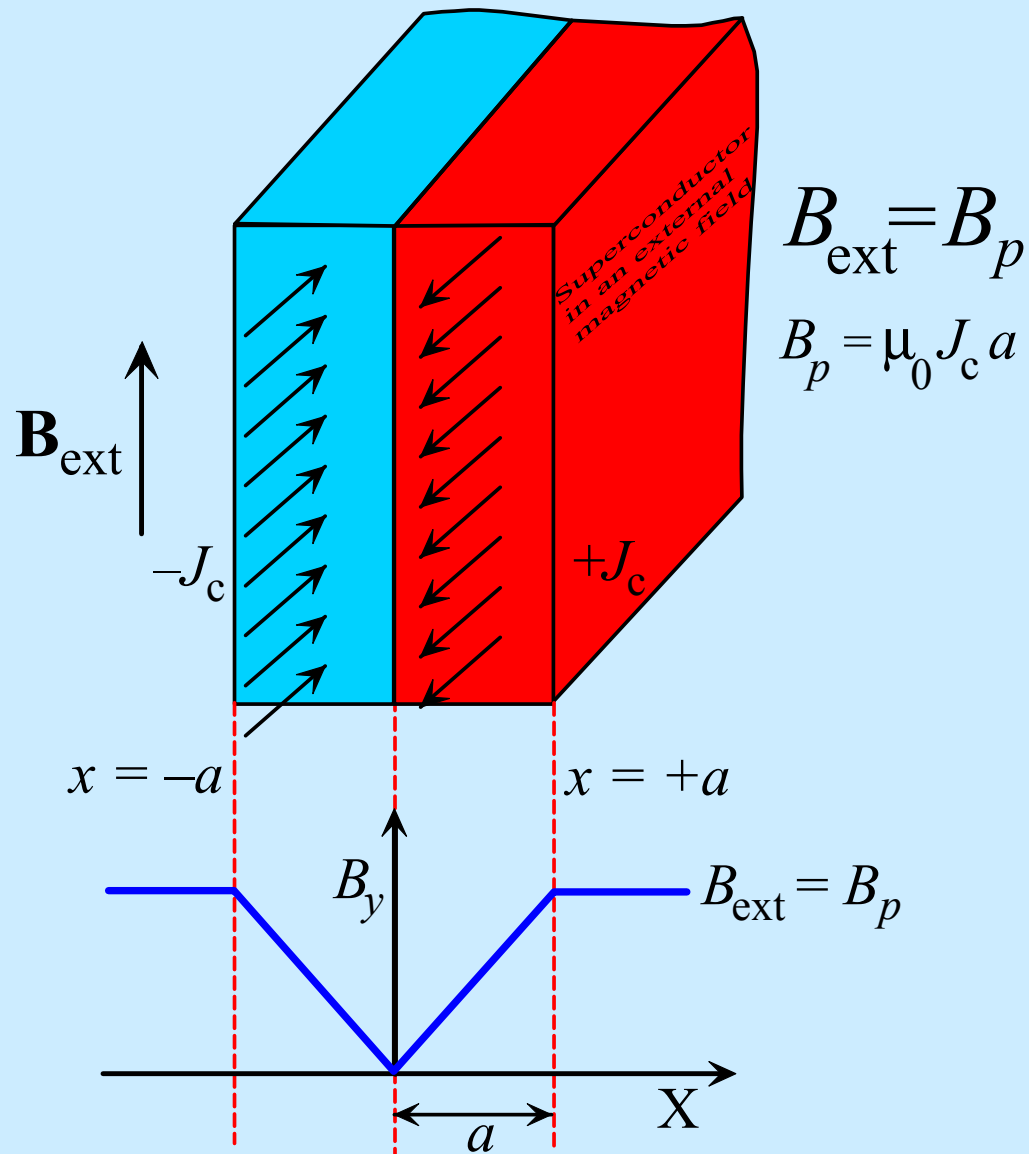
- The hysteresis in certain harmonics resulting from the ramp direction dependence of eddy currents can be understood qualitatively by a simple model known as the *Critical State Model*.
- In this model, it is assumed that eddy currents, with a density equal to the critical current density, J_c , are induced in the superconductor to counteract any change in the external field.

Superconductor in an External Field



For low external fields (less than the **Penetration Field**), persistent currents are localized within a thickness, t , sufficient to shield the interior from the external field.

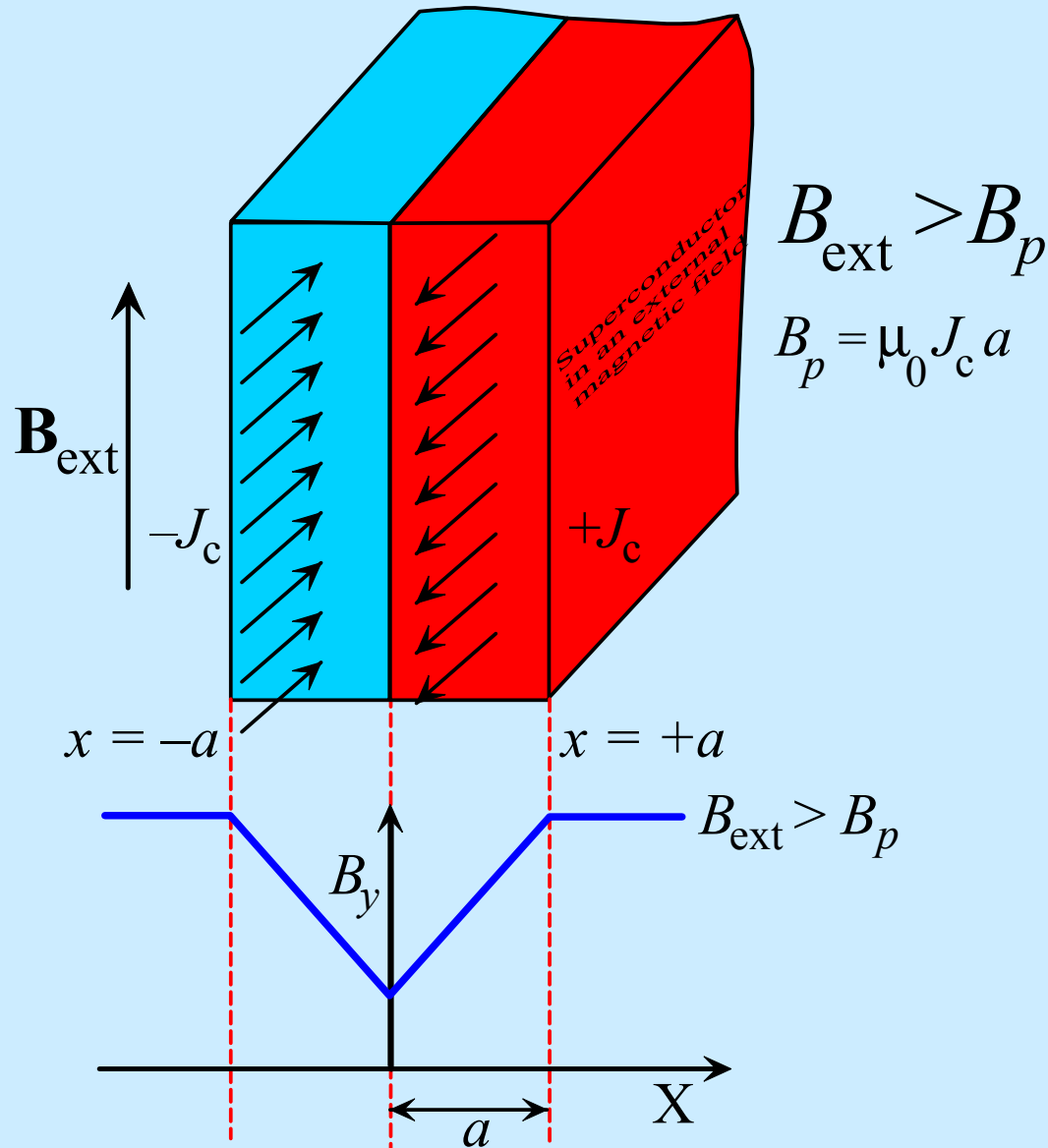
Superconductor in an External Field



At a certain field (the **Penetration Field**), persistent currents span the entire volume of the superconductor.

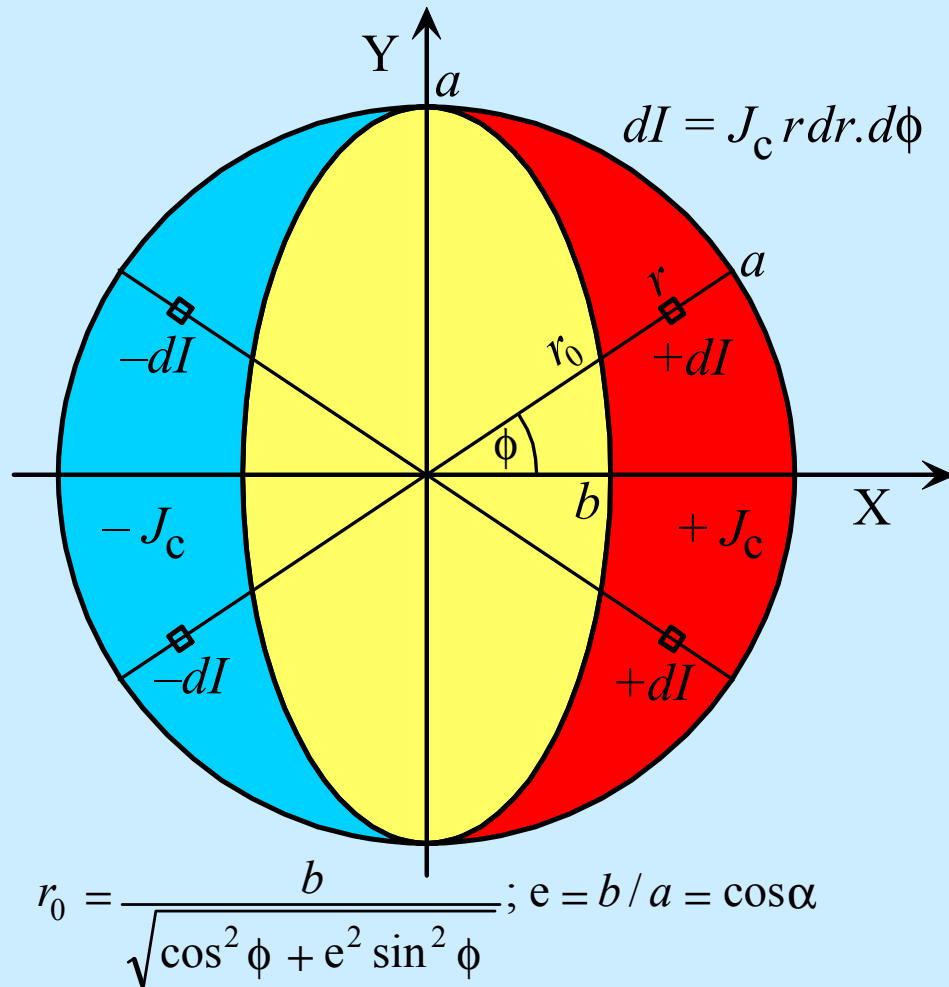
The field at the center is still zero.

Superconductor in an External Field



At external fields higher than the **Penetration Field**, persistent currents span the entire volume of the superconductor, but the interior is not completely shielded.

Filament Magnetization: 1st Up Ramp

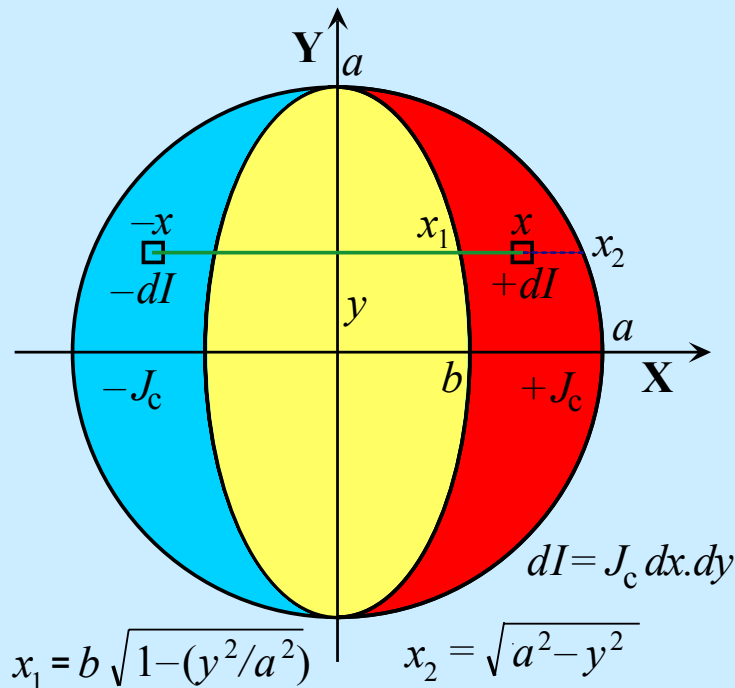


When the external field is increased from zero to a small value, B_a , shielding currents are set up such that the field inside is zero. The boundary of shielding currents may be approximated by an ellipse.

$$\left(\frac{B_a}{B_p} \right) = \left[1 - \frac{\cos \alpha}{(\sin \alpha) / \alpha} \right]$$

$$B_p = \left(\frac{2\mu_0 J_c a}{\pi} \right); \quad B_a \leq B_p; \quad 0 \leq \alpha \leq \pi/2$$

Magnetic Moment: 1st Up Ramp



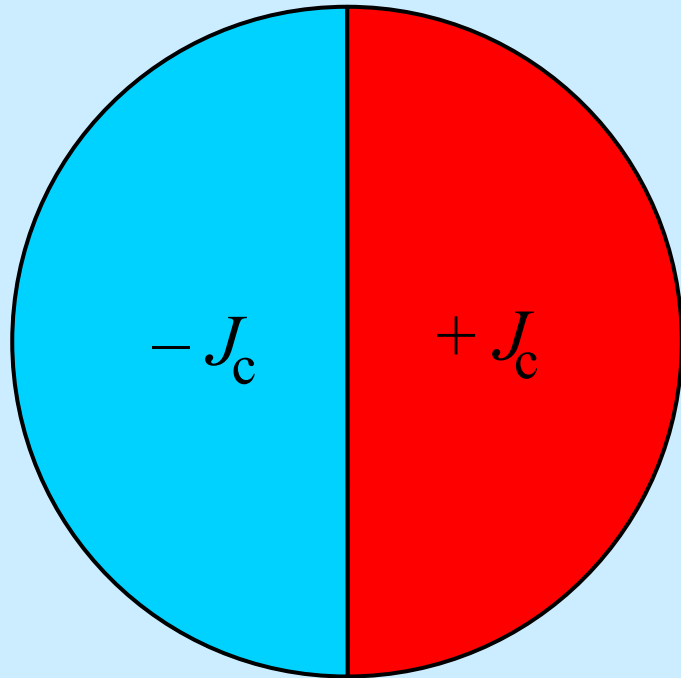
The persistent currents produce a magnetic moment. The **Magnetization**, or the magnetic moment per unit volume, can be calculated by integrating over elemental loops.

$$M = \frac{-2\mu_0 J_c}{\pi a^2} \int_{-a}^a dy \int_{x_1}^{x_2} x dx = -\left(\frac{4}{3\pi}\right) \mu_0 J_c a \left[1 - \frac{b^2}{a^2}\right]$$

$$|M_{\text{peak}}| = \left(\frac{4}{3\pi}\right) \mu_0 J_c a$$

Magnetization is proportional to the **critical current density**, and the **filament diameter**.

Full Magnetization: Up Ramp



$$B_{\text{applied}} \geq B_p$$

$$B_{\text{inside}} = B_{\text{applied}} - B_p$$

$$\left(\frac{M}{|M_{\text{peak}}|} \right) = -1$$

Example:

$$J_c = 2 \times 10^4 \text{ A/mm}^2$$

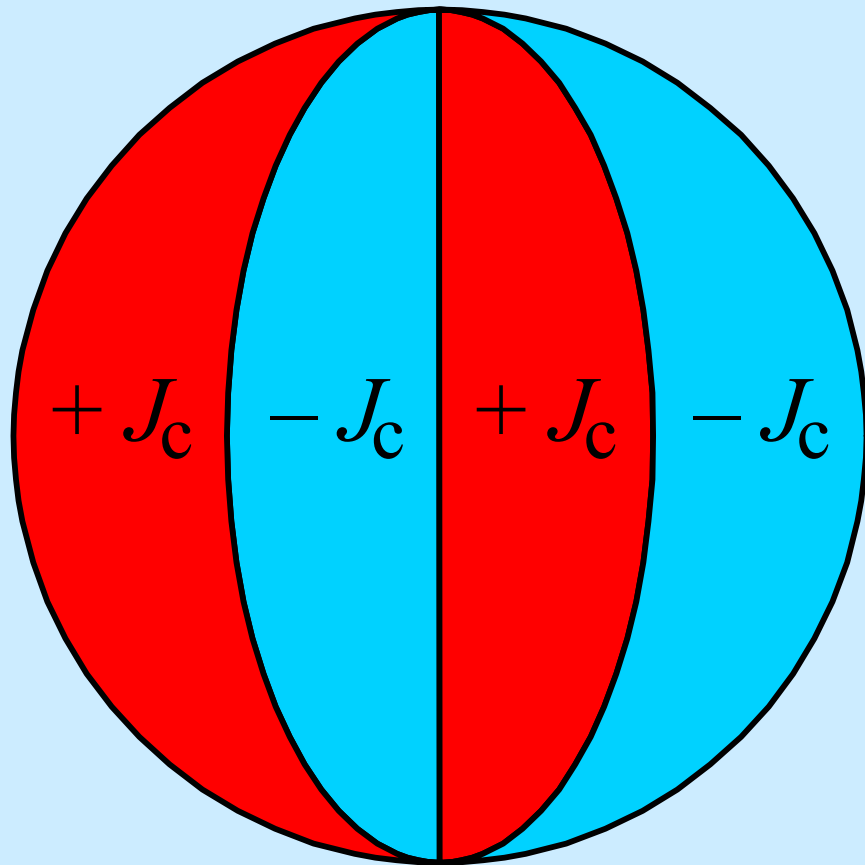
$$\text{dia.} = 2a = 6 \mu\text{m}$$

$$B_p = 0.048 \text{ Tesla}$$

$$B_p = \left(\frac{2\mu_0 J_c a}{\pi} \right); |M_{\text{peak}}| = \left(\frac{4}{3\pi} \right) \mu_0 J_c a$$

At fields above the penetration field, the filament is fully magnetized. In this simple model, the magnetization does not change as the field is increased further.

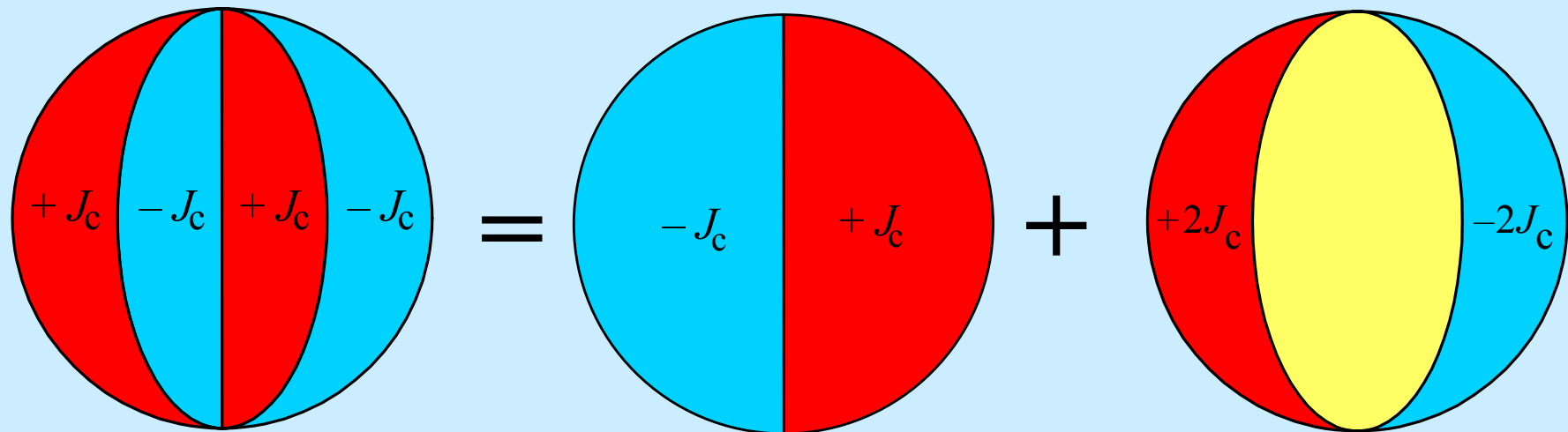
Magnetization on the Down Ramp



When the field is reduced from the maximum field by an amount B_0 , new shielding currents are induced, which are opposite in sign to the earlier currents.

The geometry of the new shielding currents is such that a field of $+B_0$ is produced inside the filament.

Magnetization on the Down Ramp



New distribution
at B_a

=

Distribution
at B_{\max}

+

New shielding currents
(Density = $2 \times J_c$)

$$B_a = B_{\max} - B_0$$

$$M / |M_{\text{peak}}| = -1$$

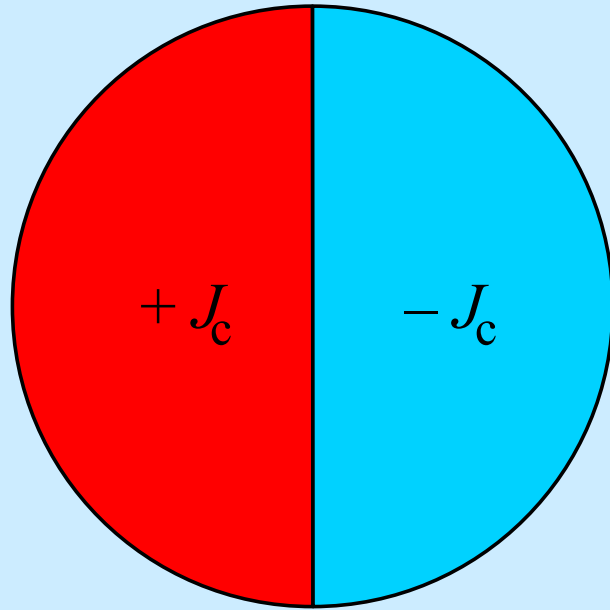
$$|M_{\text{peak}}| = \left(\frac{4}{3\pi} \right) \mu_0 J_c a$$

$$B_0 = \left(\frac{4\mu_0 J_c a}{\pi} \right) \left[1 - \frac{\cos \alpha}{(\sin \alpha) / \alpha} \right]$$

$$M = \left(\frac{8}{3\pi} \right) \mu_0 J_c a \sin^2 \alpha$$

$$\cos \alpha = b / a$$

Full Magnetization: Down Ramp

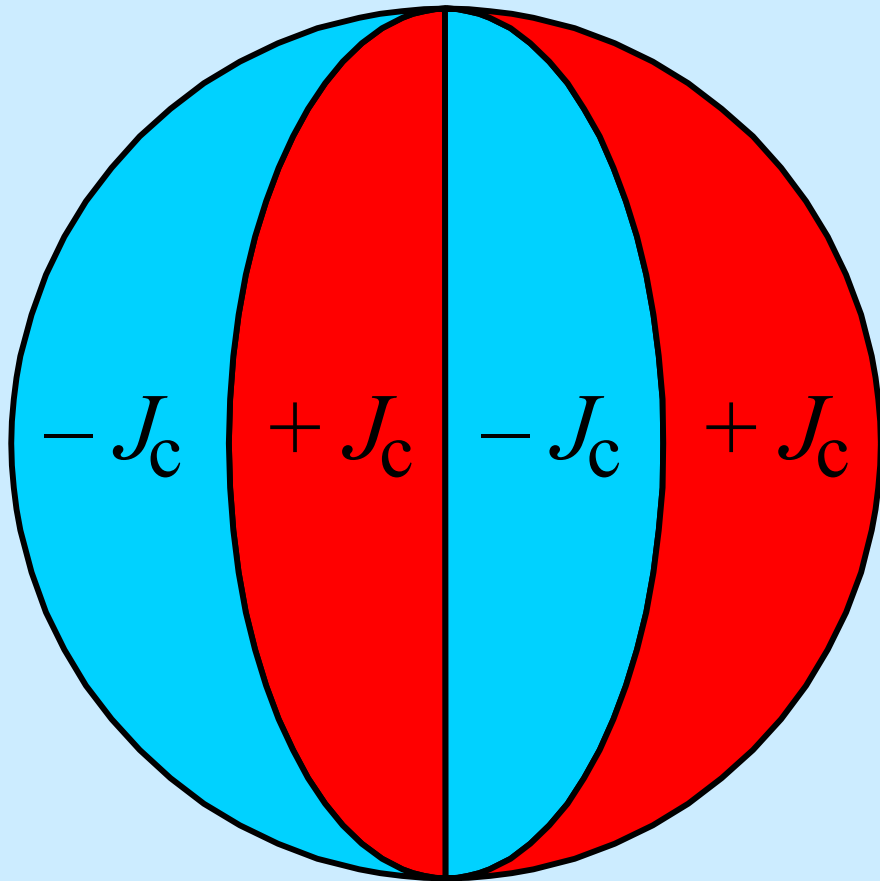


$$B_a \leq B_{\max} - 2B_p$$
$$\left(\frac{M}{|M_{\text{peak}}|} \right) = +1$$

Starting from an unmagnetized state, the filament ends up with a magnetization of $+M_{\text{peak}}$ after a cycle to B_{\max} and back.

As the field is reduced from a certain maximum value, B_{\max} , the superconducting filaments are fully magnetized again in the opposite direction. This happens at a field of $B_{\max} - 2B_p$. This continues until the minimum field, B_{\min} , as long as the direction of field change is not reversed.

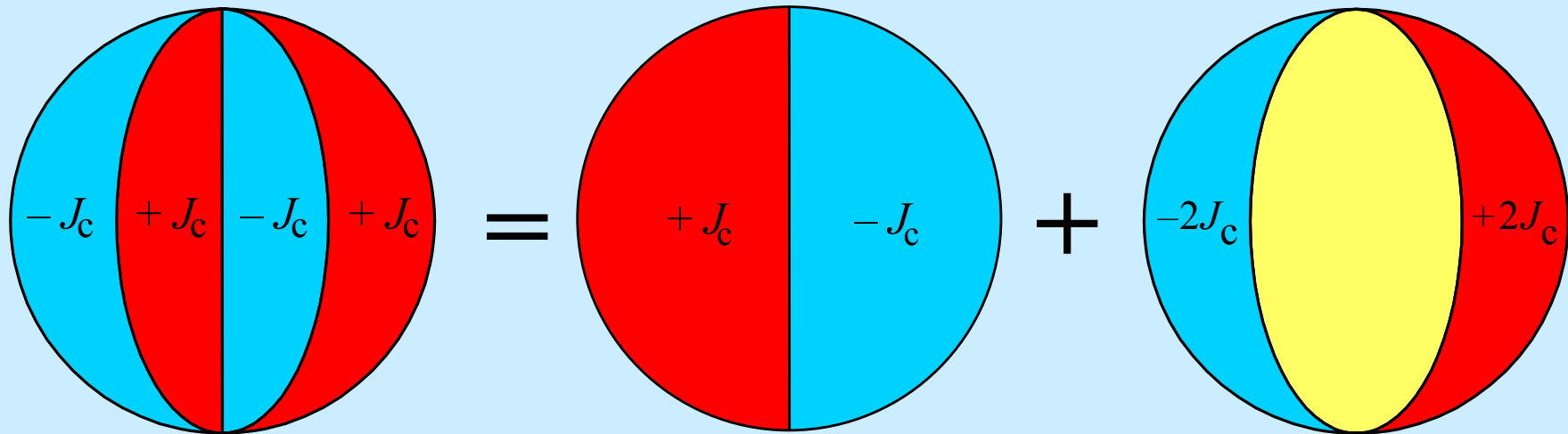
Magnetization on the 2nd Up Ramp



If the field is increased again after reaching B_{\min} , new shielding currents are induced, which are opposite in sign to the earlier currents.

If the applied field is $B_{\min} + B_0$, the geometry of the new shielding currents is such that a field of $-B_0$ is produced inside the filament.

Magnetization on the 2nd Up Ramp



New distribution
at $B_a > B_{\min}$

=

Distribution
at B_{\min}

+

New shielding currents
(Density = $2 \times J_c$)

$$B_a = B_{\min} + B_0$$

$$M / |M_{\text{peak}}| = +1$$

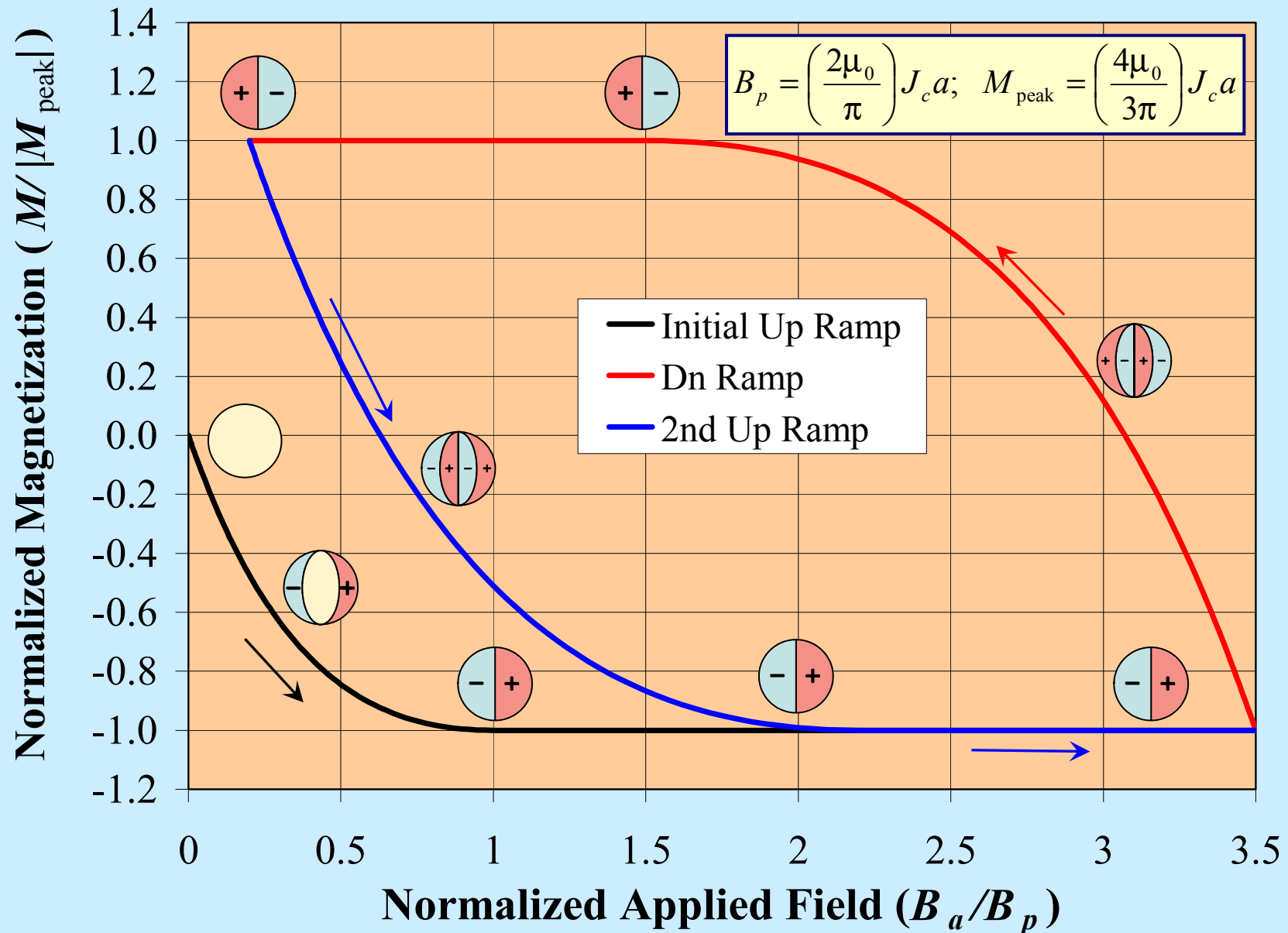
$$|M_{\text{peak}}| = \left(\frac{4}{3\pi} \right) \mu_0 J_c a$$

$$B_0 = \left(\frac{4\mu_0 J_c a}{\pi} \right) \left[1 - \frac{\cos \alpha}{(\sin \alpha) / \alpha} \right]$$

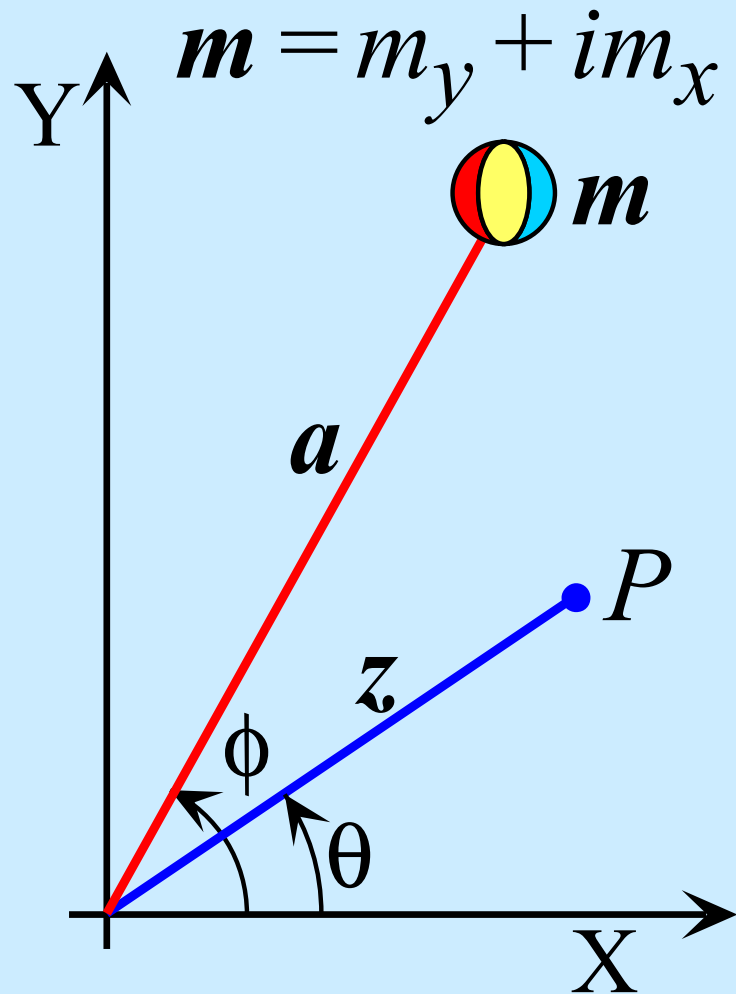
$$M = - \left(\frac{8}{3\pi} \right) \mu_0 J_c a \sin^2 \alpha$$

$$\cos \alpha = b / a$$

Magnetization Vs. Applied Field



Harmonics due to a Magnetic Moment



Field at the point P is given by:

$$B(z) = -\left(\frac{\mathbf{m}^*}{2\pi a^2}\right)\left[1 - \left(\frac{z}{a}\right)\right]^{-2}$$

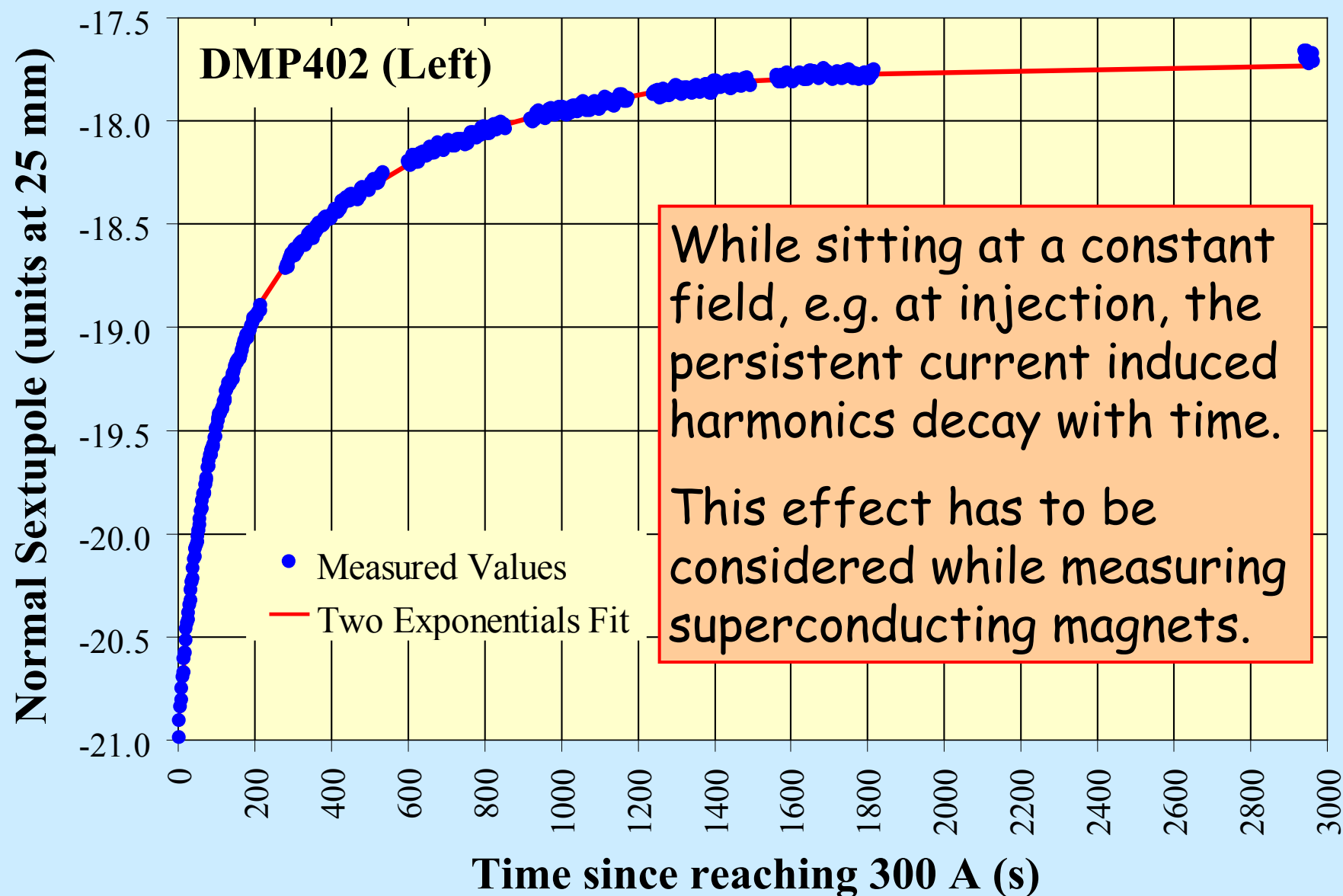
$$\equiv \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}}\right)^{n-1}$$

Harmonics are thus given by:

$$B_n + iA_n$$

$$= -\left(\frac{\mathbf{m}^* n}{2\pi a^2}\right) \left(\frac{R_{ref}}{a}\right)^{n-1} \exp[-i(n+1)\phi]$$

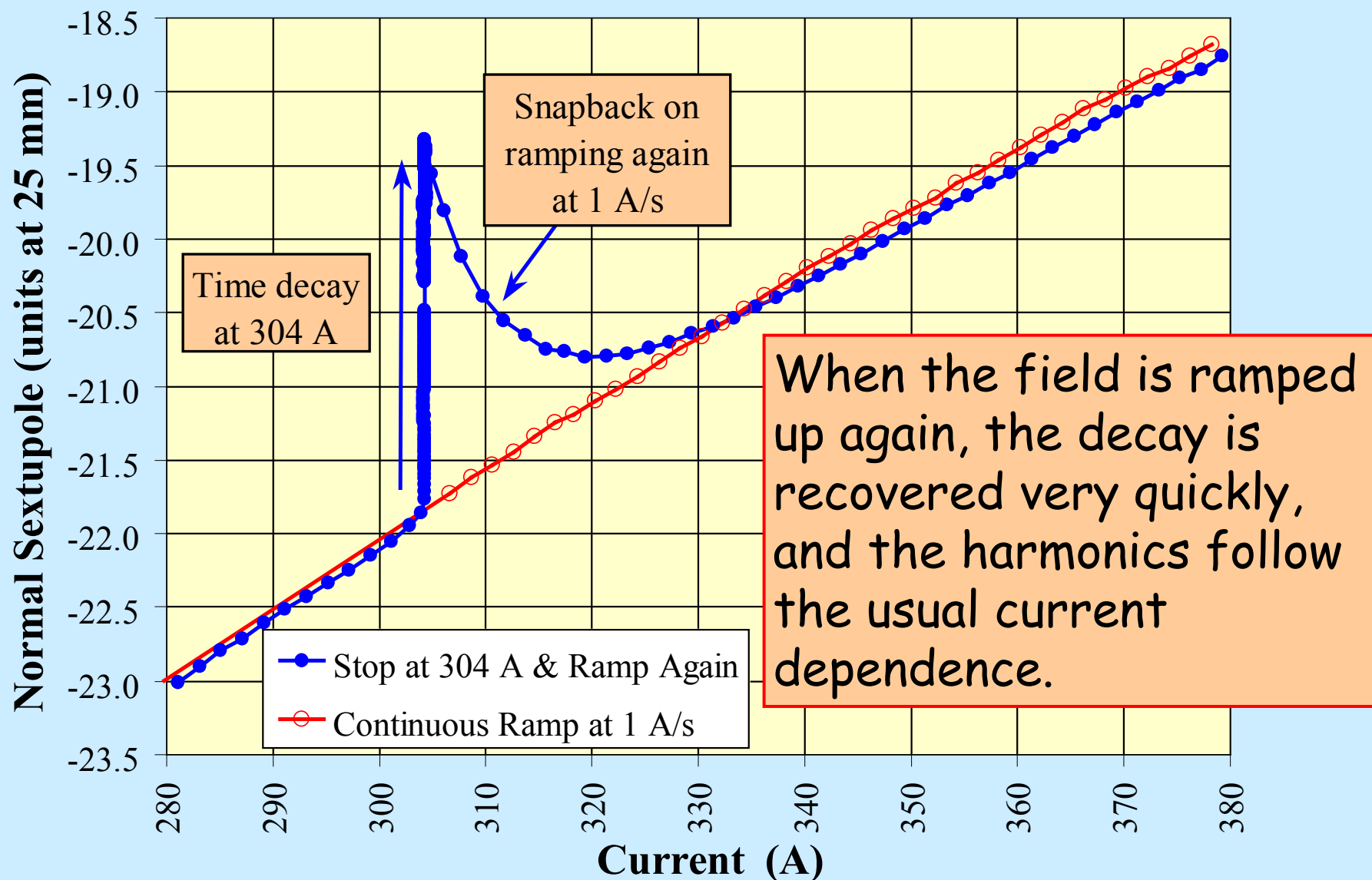
Time Decay of Harmonics



Time Decay of Harmonics

- **Flux Creep:** is caused by thermal activation and Lorentz forces due to transport current, effectively reducing J_c with time. This is temperature dependent, and produces a logarithmic decay.
- **Boundary Induced Coupling Currents:** Different strands in a cable may carry different currents (e.g. due to spatial variations in time derivative of the field). These coupling currents produce periodic axial variation of the field. As the variations decay, regions which were at a higher field jump to the down ramp branch. Can vary magnet to magnet.

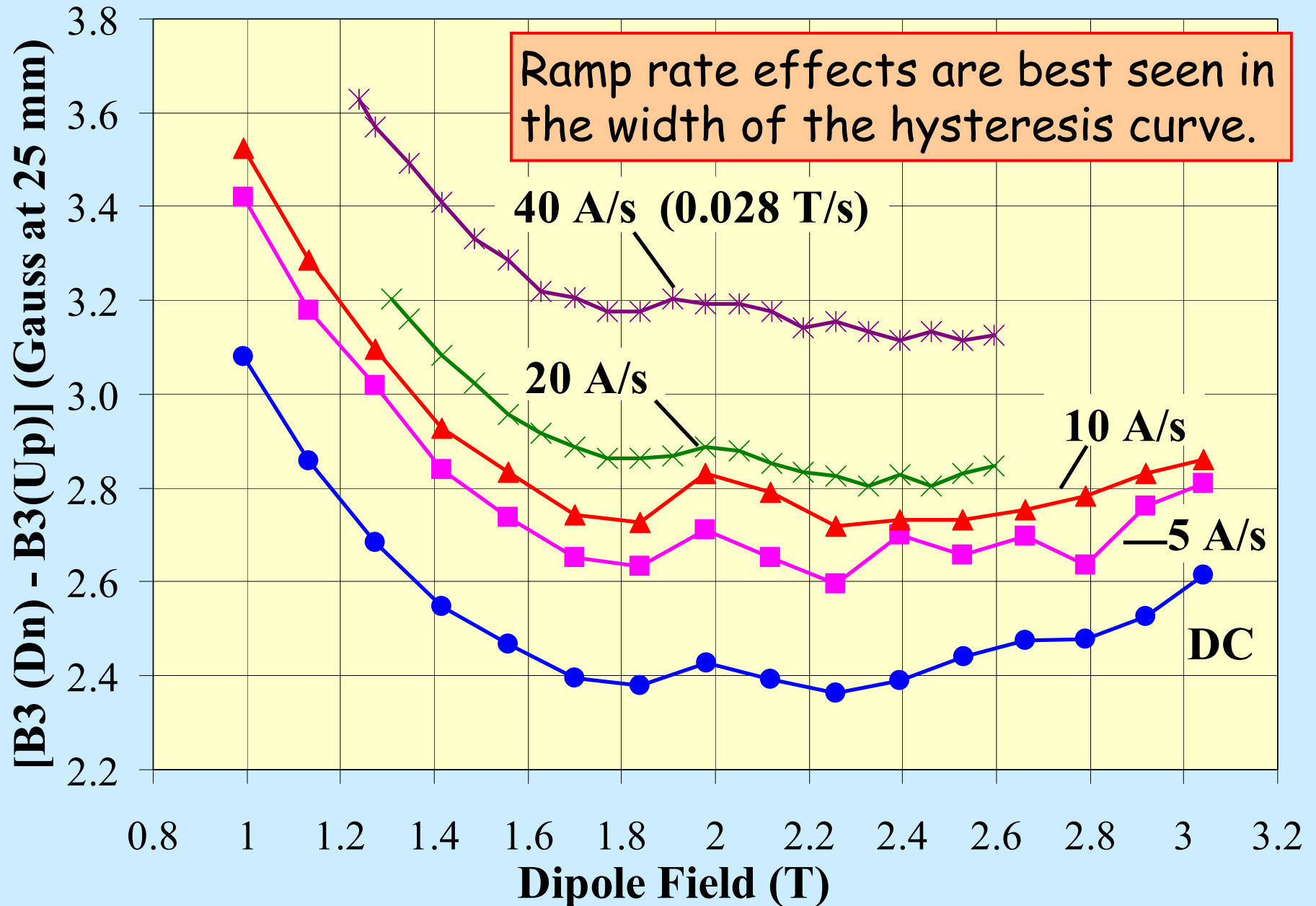
Snapback of Harmonics



Ramp Rate Effects

- The field quality is typically measured with rotating coils under DC excitation.
- If the magnet is being ramped at a high ramp rate, eddy currents can cause significant distortion of the field, and thus generate harmonics.
- The extent of distortion depends on the ramp rate and inter-strand resistance in multi-strand cables.

D1L103: Ramp Rate & Magnetization Effects



Measurements of Dynamic Effects

- The harmonics decay rather rapidly in the beginning.
- The snapback occurs within a few seconds.
- The primary field is also changing with time, particularly when the current is continuously being ramped at a high rate.
- All these factors present unique challenges in the measurement of dynamic effects. The key measurement issue is *time resolution*.

Techniques for “Fast” Measurements

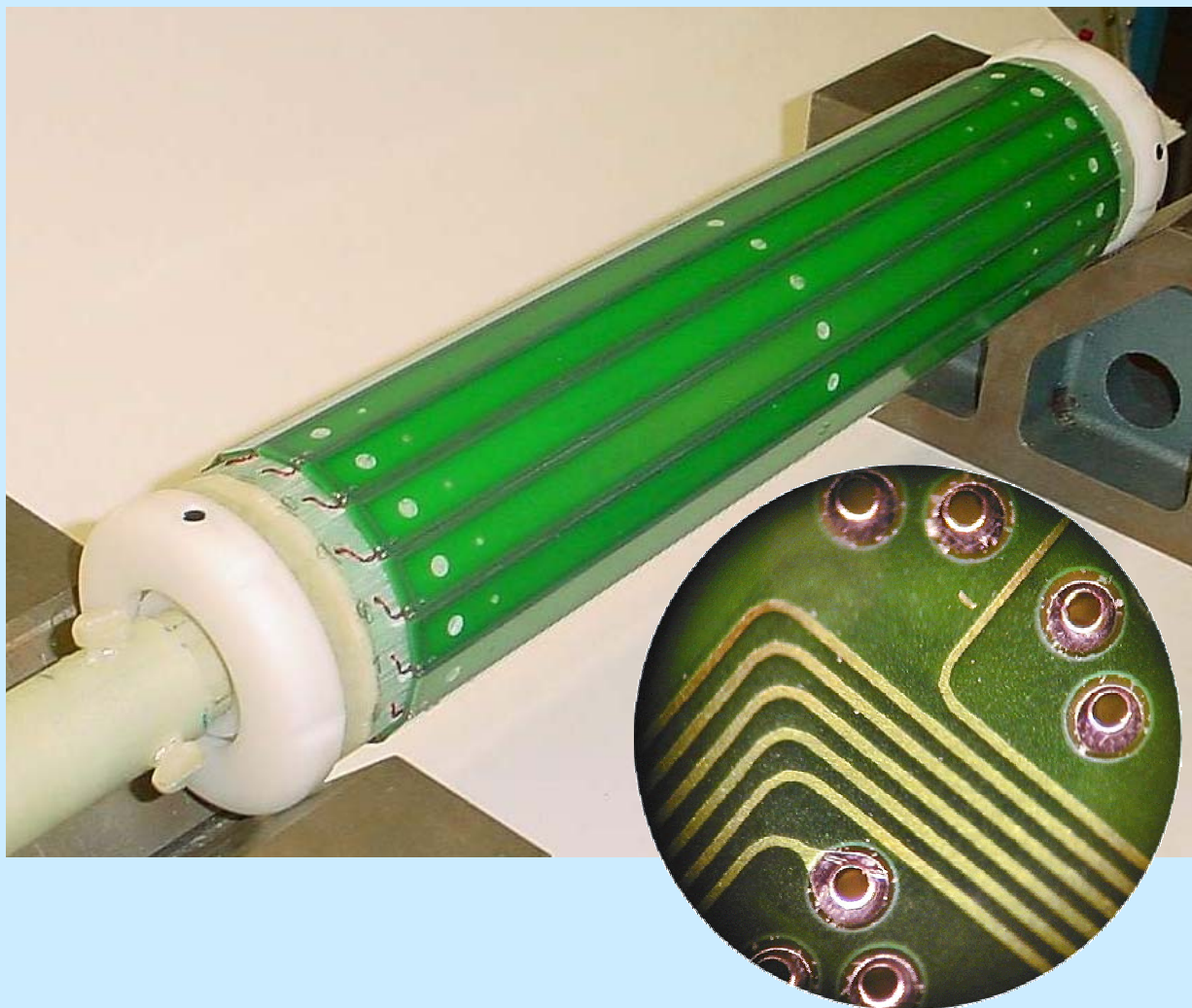
- Rotate a harmonic coil as fast as practical to improve time resolution. This allows measurements with ~ 1 s typical resolution. *(OK for time decay and snapback studies)*
- The same technique can be used to measure harmonics during very slow current ramps.
- The technique can be extended to somewhat higher ramp rates by refining the analysis.
- This method is impractical for very high ramp rates, or very short time scales.

Techniques for “Fast” Measurements

- One could use *non-rotating probes* to overcome the time resolution problem.
- Without a rotating probe, one needs a *multiple probe system* to get harmonic information.
- A system of 3 *Hall probes*, for example, can measure the sextupole component. Similarly, *NMR arrays* have been built with many probes.
- Intercalibration of individual probes and non-linear behavior are some of the problems that must be addressed in such techniques.

A Harmonic Coil Array

(Under development at BNL)



16 Printed Circuit Windings, 10 layers

6 turns/layer

Nominally identical windings due to printed circuits

Non-rotating coil for very fast measurements of harmonics (~ 50 Hz)

Summary

- Persistent currents in superconductors produce history dependent harmonics.
- These harmonics decay with time, but also snapback as soon as the ramp is resumed.
- These dynamic effects demand particular care in the measurements of superconducting magnets. Good time resolution is also required.
- Various techniques have been employed for dynamic measurements, but no single technique seems to be the “best”. (Promising R&D area.)